SOLUTION BY THE PROPOSER.

Let $\sin A = a$, or $A = \sin^{-1} a$, and let $b = \sin (A + nh) = \sin B$. Hence $n = (B - A)/h$.

Then we have

$$\begin{array}{cccc} a & & & b \\ \sin A & \sin (A + h) \cdots & \sin [A + (n - 1)h] & \sin (A + nh) \end{array}$$

$$\begin{aligned}
\int_a^b \sin^{-1} x dx &= \lim_{h \doteq 0} [\{\sin (A + h) - \sin A\} \sin^{-1} \sin A \\
&\qquad + \{\sin (A + 2h) - \sin (A + h)\} \sin^{-1} \sin (A + h) + \cdots \\
&\qquad + \{\sin (A + nh) - \sin [A + (n - 1)h]\} \sin^{-1} \sin [A + (n - 1)h]] \\
&= \lim_{h \doteq 0} [- h\{\sin (A + h) + \sin (A + 2h) + \cdots + \sin (A + nh)\} - A \sin A \\
&\qquad + (A + nh) \sin (A + nh)] \\
&= \lim_{h \doteq 0} \left[- \frac{h/2}{\sin h/2} \left\{ \cos \left(A + \frac{h}{2} \right) - \cos \left[A + (2n + 1) \frac{h}{2} \right] \right\} - A \sin A \right. \\
&\qquad \left. + (A + nh) \sin (A + nh) \right] \\
&= \lim_{h \doteq 0} \left[- \frac{h/2}{\sin h/2} \left\{ \cos \left(A + \frac{h}{2} \right) - \cos \left(B + \frac{h}{2} \right) \right\} - A \sin A + B \sin B \right] \\
&= \cos B - \cos A + B \sin B - A \sin A \\
&= \sqrt{1 - b^2} - \sqrt{1 - a^2} + b \sin^{-1} b - a \sin^{-1} a,
\end{aligned}$$

which agrees with the result given in the tables.

An excellent solution was also received from A. M. HARDING.

367. Proposed by C. N. SCHMALL, New York City.

Show that the volume inclosed by the surface $(x^2 + y^2 + z^2)^5 = (a^3x^2 + b^3y^2 + c^3z^2)^2$ is $\frac{4}{9}\pi(a^3 + b^3 + c^3)$.

SOLUTION BY A. M. HARDING, University of Arkansas, AND A. R. NAUER, St. Louis, Mo.

Let $x = r \sin \varphi \cos \theta$, $y = r \sin \varphi \sin \theta$, $z = r \cos \varphi$. Substituting in the given equation, we obtain

$$r^3 = a^3 \sin^2 \varphi \cos^2 \theta + b^3 \sin^2 \varphi \sin^2 \theta + c^3 \cos^2 \varphi.$$

Then the volume inclosed by the surface is given by

$$\begin{aligned}
V &= 4 \int_0^{\pi/2} \int_0^{\pi} \int_0^r r^2 \sin \varphi dr d\theta d\varphi, \\
&= \frac{4}{3} \int_0^{\pi/2} \int_0^{\pi} (a^3 \sin^2 \varphi \cos^2 \theta + b^3 \sin^2 \varphi \sin^2 \theta + c^3 \cos^2 \varphi) \sin \varphi d\theta d\varphi, \\
&= \frac{4}{3} \int_0^{\pi/2} \left(\frac{\pi}{2} \cdot a^3 \sin^3 \varphi + \frac{\pi}{2} b^3 \sin^3 \varphi + \pi c^3 \cos^2 \varphi \sin \varphi \right) d\varphi, \\
&= \frac{4\pi}{3} \left[\frac{a^3}{2} \cdot \frac{2}{3} + \frac{b^3}{2} \cdot \frac{2}{3} + \frac{c^3}{3} \right] = \frac{4\pi}{9} (a^3 + b^3 + c^3).
\end{aligned}$$